Privacy-Preserving Strategy for Negotiating Stable, Equitable and Optimal Matchings

Maxime Morge and Gauthier Picard

Abstract The assignment problem has a wide variety of applications and in particular, it can be applied to any two-sided market. In this paper, we propose a multi-agent framework to distributively solve this kind of assignment problems, by providing agents which negotiate with respect to their preferences. We present here a realisation of the minimal concession strategy. Our realisation of the minimal concession strategy has useful properties: it preserves the privacy and improves the optimality of the solution and the equity amongst the partners.

1 Introduction

Negotiation over the assignments of agents is a new challenging area [10]. This problem has the potential for attracting interests, as resource allocation [5], from microeconomics and social choice theory on the one hand and computer science and AI on the other. The assignment problem has a wide variety of practical applications and in particular, it can be applied to any two-sided market: students/projects, carpool, home swapping, service provider/requesters, etc.

A particular instantiation of the assignment problem consists of the *stable marriage problem* (SMP) which it is commonly stated as mapping between two communities (e.g. men and women). In this paper, we propose a multi-agent framework to distributively solve this kind of problems, by providing agents which negotiate with respect to their preferences. Here assignments are viewed as emergent phenomena resulting from local agent negotiations. The objective of such procedure is to find an assignment that is *optimal*. For this purpose, we can consider different notions of *social welfare*. Within this paper, we propose *Casanova*, a distributed method

Gauthier Picard
ISCOD team, Ecole des Mines de Saint-Etienne, e-mail: picard@emse.fr
Maxime Morge
SMAC team, Laboratoire d'Informatique Fondamentale de Lille, e-mail: Maxime.Morge@lifl.fr

to solve SMP. We seek to provide agent behaviors leading negotiation processes to socially optimal assignments. We propose a realisation of the minimal concession strategy applied to SMP. Our strategy has useful properties: it preserves the privacy, it improves the optimality of the solution and the equity amongst partners.

2 Stable Marriage Problem

SMP were first studied by [6] in order to find optimal assignments. In a SMP there are two finite sets of participants: the set of men and the set of women.

Definition 1 (SM). A **stable marriage problem** of size n (with $n \geq 1$) is a couple $SM = \langle X, Y \rangle$ where:

- $X = \{x_1, \ldots, x_n\}$ is a set n men ranking women in a strict and complete order forming his preference list. $\forall 1 \leq i \leq n,\ x_i = (y_i^0, \ldots, y_i^{n-1})$
- $Y = \{y_1, \ldots, y_n\}$ is a set n women ranking men in a strict and complete order forming her preference list. $\forall 1 \leq i \leq n,\ y_i = (x_i^0, \ldots, x_i^{n-1})$

A person z_1 prefers a partner t_2 to another partner t_3 if and only if t_2 precedes t_3 on z_1's preference list (denoted $t_2 \succ_{z_1} t_3$).

A matching is just a complete one-to-one mapping between the two sexes such that a man x is mapped to a woman y if and only if y is mapped to x.

Definition 2 (Matching). Let $SM = \langle X, Y \rangle$ be a stable marriage problem of size n (with $n \geq 1$). A **matching** for SM is a n-uplet $M = \langle m_1, \ldots, m_n \rangle$ of n marriages where each m_i (with $1 \leq i \leq n$) is a couple $(x_i, y_i) \in X \times Y$ such that the matching is complete, i.e. each individual is married. Formally, $\forall x \in X\ \exists! y \in Y\ (x, y) \in M$. The partner of the agent z in accordance with the matching M is denoted $p_M(z)$.

We want to marry men and women together such that there are no two people of opposite sex who would both rather have each other than their current partners, i.e. finding a stable matching.

Definition 3 (Stable matching). Let $SM = \langle X, Y \rangle$ be a stable marriage problem of size n (with $1 \geq n$). and M a matching for SM. M is **stable** iff:
$\forall (x_i, y_i) \in M\ \nexists (x_j, y_j) \in M\ x_j \succ_{y_i} x_i$ and $y_j \succ_{x_i} y_i$.

A typical objective in SM is to find an assignment that is optimal with respect to a metric that depends on the preferences of the agents. For this purpose, we assume that individual agents evaluate their satisfaction using utility functions mapping assignments to numerical values.

Definition 4 (Utility function). Let $SM = \langle X, Y \rangle$ be a stable marriage problem of size n (with $n \geq 1$), $z = (t_i^0, \ldots, t_i^k, \ldots, t_i^{n-1})$ an individual agent and T be the potential partners of z. The **utility function** of the agent z is a function $u_z : T \rightarrow \mathbb{R}$. If the matching assigns z with t_i^k, then $u_z(t_i^k) = \frac{(n-1)-k}{n-1}$.

The social welfare theory is used to evaluate the matching, considering the welfare of each person [1]. In this study, we derive from this theory four notions adapted for the stable marriage problem.

Definition 5 (Social welfare). Let $SM = \langle X, Y \rangle$ be a stable marriage problem of size n (with $n \geq 1$) and M a matching for SM.

- The **utilitarian welfare** considers the welfare of the whole society: $sw_u(X \cup Y) = \Sigma_{z \in X \cup Y} u_z(p_M(z))$.
- The **male welfare** considers the welfare of the men: $sw_u(X) = \Sigma_{x \in X} u_x(p_M(x))$.
- The **female welfare** considers the welfare of the women: $sw_u(Y) = \Sigma_{y \in Y} u_y(p_M(y))$.
- The **equity welfare** considers the fairness among partners' welfare in every marriage: $sw_e(X \cup Y) = 1 - \frac{|sw_u(X) - sw_u(Y)|}{n}$.

Utilitarian welfare can be used to measure the quality of a matching from the viewpoint of the system as a whole. The equity welfare may be a suitable indicator when we have to satisfy both the men and the women.

Gale and Shapley described in [6] a centralized algorithm (GS) that always finds a stable matching for any instance of the SMP. They also noted that this algorithm produces a matching in which each man has the best partner he can have in any stable matching. GS involves a sequence of proposals from men to women. It starts by setting all persons free. GS iterates until all the men are engaged. Each man x always proposes marriage to his most-preferred woman, y. When y is already married (e.g. with x_2) she discards the previous proposal with x_2 and x_2 is set free. Afterwards, x and y are engaged to each other. Woman y deletes from her preference list each man x_3 that is less preferred than x. Conversely, man x_3 deletes y from his preference list. Finally, if there is still a free man a new proposal is started. Otherwise, the algorithm terminates. This algorithm is commonly known as the men-propose algorithm because it can be expressed as a sequence of "proposals" from the men to the women. [6] established the existence of a stable marriage thanks to GS that constructs a men-optimal (resp. women-optimal) stable matching, i.e. it optimizes the *male welfare* (resp. *women welfare*).

Example 1. Let us consider the SM $\langle X, Y \rangle$ of size 3:

$x_1 = (y_2, y_1, y_3)$ $\qquad$ $y_1 = (x_2, x_1, x_3)$
$x_2 = (y_3, y_2, y_1)$ $\qquad$ $y_2 = (x_3, x_2, x_1)$
$x_3 = (y_1, y_3, y_2)$ $\qquad$ $y_3 = (x_1, x_3, x_2)$

The output of the men-propose GS algorithm is $M_1 = \langle (x_3, y_1), (x_1, y_2), (x_2, y_3) \rangle$. In accordance with M_1, $sw_u(X \cup Y) = 3$, $sw_u(X) = 3$, $sw_u(Y) = 0$ and $sw_e(X \cup Y) = 1$. We can notice that a stable matching exists even if it is not found by the GS algorithms: $M_3 = \langle (x_1, y_1), (x_2, y_2), (x_3, y_3) \rangle$. In accordance M_3, $sw_u(X \cup Y) = 3$, $sw_u(X) = 1.5$, $sw_u(Y) = 1.5$ and $sw_e(X \cup Y) = 0$.

A distributed version of the GS algorithm (DisEGS) has been proposed by [3]. Each man (and woman) is represented by an agent which exchange messages (*propose*, *accept* and *delete*) as to reproduce the GS algorithm and find the same stable assignment. Contrary to classical GS, each agent keeps its own preferences, which represents a interesting step towards privacy.

3 Casanova Algorithm

In this study, we consider matchings as emergent phenomena resulting from local agent negotiations. The Casanova algorithm is a negotiation strategy to reach a matching in a SMP. Contrary to DisEGS, we do not distinguish men and women. Both men and women send concurrently proposals and reply with acceptance or rejections, which represents the main difficulty of this study.

According to Casanova, agents start the negotiation with the best potential partners. During the negotiation, an agent concedes minimally as soon as its optimal partners has refused. A concession is minimal for an agent since there is no other preferred partner which has not yet refused.

The strategy starts by setting the agent free and the concession level equals to 0. At each run, the agent starts by sending proposals to the sub-list of agents corresponding its concession level. During the first step, the agent sends a *proposal* to the optimal partner. During the second run, the agent addresses proposals to the two preferred agents, and so on. When the agent receives a *proposal*, it only accepts the ones corresponding to its concession level, called acceptable proposals. In this case, the agent gets divorced with its current partner if it is required and it gets engaged with its new partner. It is worth noticing that the agents are allowed to divorce for a preferred partner if and only if the agent is not engaged but married (in order to avoid deadlock). When the agent receives an *acceptance*, the agent *confirms* or withdraws depending whetever or not its current partner is the sender of the acceptance. As previously, the agent is allowed to divorce if he has some regrets, i.e. the potential partner is preferred to the current partner. When the agent receives a *withdrawal*, the agent get divorced. We can notice that the agents count the response to its proposals. If all of them are received and the agent is still free, it must concede, i.e. go further in its preference list to add acceptable partners. When the agent receives a *divorce* notification, the agent takes it into account.

Casanova outputs a stable matching. Suppose it is not the case, i.e. there is an agent A that prefers an agent B (that it's not matched to) and at the same time B also prefers A over the one B is matched with. According to the concession level of A (resp. B), A would propose to (resp. B would accept) B (resp. A) for a partnership.

Example 2. Let us consider the Casanova strategy implemented by the multi-agent system set up as in Ex. 1. As a reminder, y_3's preferences are (x_1, x_3, x_2). Initially, y_3 is free and her concession level is equal to 0. So, the only acceptable partner is x_1. In our example, the local negotiations lead to the stable matching $M_3 = \langle(x_1,y_1),$ $(x_2,y_2),(x_3,y_3)\rangle$ such that $sw_e(X \cup Y) = 0$.

4 Evaluation

Casanova has been implemented with Jason [2]. In order to evaluate Casanova, we run it for some random SMP instances [7] where *n*, the number of potential partners, is between 2 and 100 (see. Fig. 1). For each instance of SMP, we run 10 times Casanova. Firstly, we compare the value of the equity welfare, the male welfare and the utilitarian welfare with the one obtained with the help of the GS (or DisEGS) algorithms. Secondly, we counts the number of messages received by each agent. First, we observe that the output of Casanova is a stable marriage which is more equitable and more optimal (from the viewpoint of the system as a whole) that the one returned by the GS or DisEGS algorithms but it less optimal from the from the viewpoint of the men. Additionally, our preliminary results show that the number of messages received by each agent is linear with respect to the size of the problem.

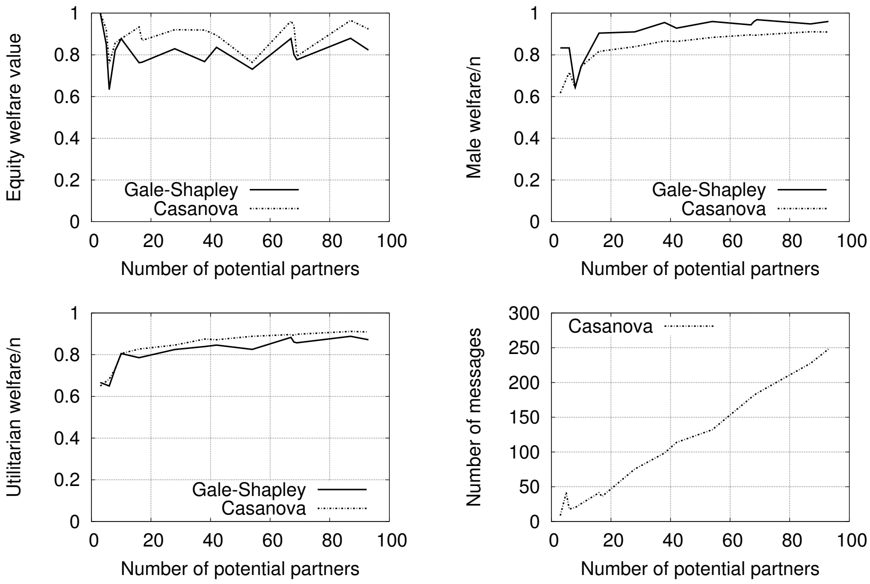

Fig. 1 Comparison of Casanova to GS (or DisEGS) results.

5 Related Works

The principle of Casanova is based on the minimal concession strategy [9, 8, 4]. Each agent starts from the partner that is best for it and, if this latter refuses, the agents concedes by considering less preferred potential partners. Differently from the game-theoretical approach [9], our approach does not assume that the agent knows the preferences of the latter [8]. We say that a proposal is a minimal concession since there is no other proposals which are preferred. Contrary to [8, 4], the deployment of the minimal concession in this paper is not limited to a bilateral negotiation. Finally, we apply the Occam's razor since we do not employ argumentation-based reasoning but a simpler reasoning.

6 Conclusion

In this paper we have presented a realisation of the minimal concession strategy applied to the SMP. According to this strategy, agents start the negotiation with their preferred partners. During the negotiation, an agent concede minimally as soon as its optimal partners has refused. A concession is minimal for an agent since there is no preferred partner which has not yet refused. Our realisation of the minimal concession strategy has useful properties. Firstly, it preserves the privacy since the agents do not reveal explicitly their preferences. Secondly, the approach improves the optimality of the matching and its equity.

We need to realize more experiments for evaluating other metrics of social welfare and for comparing with other MAS approaches.

Acknowledgements We would like to thank Bruno Beaufils, Jomi Fred Hubner, Antoine Nongaillard, and Santiago Villareal for their help on this work.

References

1. Arrow, K., Sen, A., Suzumura, K.: Handbook of Social Choice and Welfare. Elsevier (2002)
2. Bordini, R.H., Hubner, J.F., Wooldridge, M.: Programming Multi-Agent Systems in AgentSpeak Using Jason. John Wiley & Sons Ltd (2007)
3. Brito, I., Meseguer, P.: Distributed stable marriage problem. In: 6th Workshop on Distributed Constraint Reasoning at IJCAI 05, pp. 135–147 (2005)
4. Bromuri, S., Urovi, V., Morge, M., Toni, F., Stathis, K.: A multi-agent system for service discovery, selection and negotiation (demonstration). In: 8th International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS 2009), pp. 1395–1396. IFAAMAS (2009)
5. Chevaleyre, Y., Dunne, P.E., Endriss, U., Lang, J., Lematre, M., Maudet, N., Padget, J., Phelps, S., Rodrguez-Aguilar, J.A., Sousa, P.: Issues in multiagent resource allocation. Informatica **30**, 3–31 (2006)
6. Gale, D., Shapley, L.S.: College admissions and the stability of marriage. American Mathematical Monthly **69**, 9–14 (1962)
7. Gent, I.P., Prosser, P.: An empirical study of the stable marriage problem with ties and incomplete lists. In: ECAI 2002, pp. 141–145 (2002)
8. Morge, M., Mancarella, P.: Assumption-based argumentation for the minimal concession strategy. In: P. McBurney, I. Rahwan, S. Parsons, N. Maudet (eds.) Proc. of ArgMAS 2009, *LNCS*, vol. 6057, pp. 114–133. Springer Verlag, Berlin, Germany (2010)
9. Rosenschein, J.S., Zlotkin, G.: Rules of encounter: designing conventions for automated negotiation among Computers. The MIT press series of artificial intelligence. MIT press (1994)
10. Shoham, Y., Leyton-Brown, K.: Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations. Cambridge University Press (2009)