Remedial interchange

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Two questions:

- May deontic logic be relevant for the study of conversational interactions?
- Can some of the aspects of conversational interactions be described by using formal tools originally developed in the study of iterated belief change?

The dilemma of Weakening the consequent and Detachment

Consider a logic that validates both

$$\frac{\bigcirc(B/A) \qquad \Box(B \rightarrow B')}{\bigcirc(B'/A)}\ (\mathcal{WC}) \qquad \frac{\bigcirc(B/A) \qquad A}{\bigcirc B}\ (\mathcal{D})$$

Weakening the Consequent *Detachment*

Suppose the following sentences are true:

$$(1)\ \bigcirc\neg A \quad (2)\ \bigcirc(B/A) \quad (3)\ \Box(B \rightarrow A) \quad (4)\ A$$

From (2) and (4), we get $\bigcirc B$ – by $(\mathcal{D})$

From (1) and (3) we get $\bigcirc\neg B$ – by $(\mathcal{WC})$

The temporal solution (*à la* Aqvist/Hoepelman)

before the transgression $\neq$ at the time of the transgression.

Render (4) as $\oplus A$ ($\oplus$= and next). Reformulate (1)-(3) in such a way that Weakening the Consequent gives rise to the obligation $\bigcirc \oplus \oplus \neg B$, while factual detachment gives rise to the obligation $\oplus \bigcirc \oplus B$.

Pro By itself the idea is plausible enough (do not deconditionalize until the transgression has occurred!)

Con

- At the time of the transgression, the primary obligation is no longer in force:

$$A \rightarrow \Box A \rightarrow \bigcirc A$$

- What of those cases where the remedy precedes in time the transgression?

Remedial interchange

(Goffman, *Relations in Public*, 1971)

Four moves:

- `remedy` (accounts, apologies, requests)

  A: Can I use your phone to make a local call?

- `relief`, by which the victim provides a sign that the remedy offered by the offender is sufficient

  B: Sure, go ahead

- `appreciation`, by which the offender shows thankfulness

  A: That's very good of you

- `minimization`, by which the victim repeats in diminished form the relief he provided as the second move

  B: It's okay

Logical representation

	Normative premisses (α)	Additional premisses (β)
(I)	$\bigcirc \neg o$	
(II)	$\bigcirc(r_1/o)$	$\Box(r_1 \rightarrow o)$
	$\bigcirc(r_2/o \wedge r_1)$	$\Box(r_2 \rightarrow (o \wedge r_1))$
(III)	$\bigcirc(a/o \wedge r_1 \wedge r_2)$	$\Box(a \rightarrow (o \wedge r_1 \wedge r_2))$
	$\bigcirc(m/o \wedge r_1 \wedge r_2 \wedge a)$	$\Box(m \rightarrow (o \wedge r_1 \wedge r_2 \wedge a))$

with o= offence, r_2= relief, m= minimization

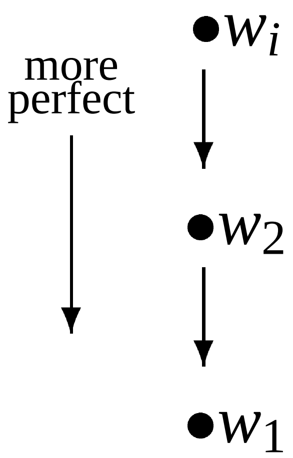
r_1= remedy, a= appreciation

Model $\mathcal{M}_1$

$w_1 : \neg o, \neg r_1, \neg r_2, \neg a, \neg m$ $\quad$ $w_2 : o, r_1, r_2, a, m$

$w_3 : o, \neg r_1, \neg r_2, \neg a, \neg m$ $\quad$ $w_4 : o, r_1, \neg r_2, \neg a, \neg m$

$w_5 : o, r_1, r_2, \neg a, \neg m$ $\quad$ $w_6 : o, r_1, r_2, a, \neg m.$



(for all $i = 3, \ldots, 6$)

Before the offence:

$$\mathcal{M}_1 \models \bigcirc \neg r_1, \mathcal{M}_1 \not\models \bigcirc r_1$$

Iterated revision (first attempt)

Natural revision (Boutilier 95). Let $\mathcal{M} = \langle W, \preceq, \iota \rangle$ be the model reflecting the initial ordering. The model obtained when $\mathcal{M}$ is "revised" by A (written $\mathcal{M}^\star A$) is $\langle W, \preceq', \iota \rangle$ with $\preceq'$ defined as follows:

$(\mathscr{P}_1)$ If $w_1 \in \min_{\mathcal{M}}(A)$ then:

$$w_1 \preceq' w_2 \text{ for all } w_2 \in W \text{ and} \quad \text{(a)}$$

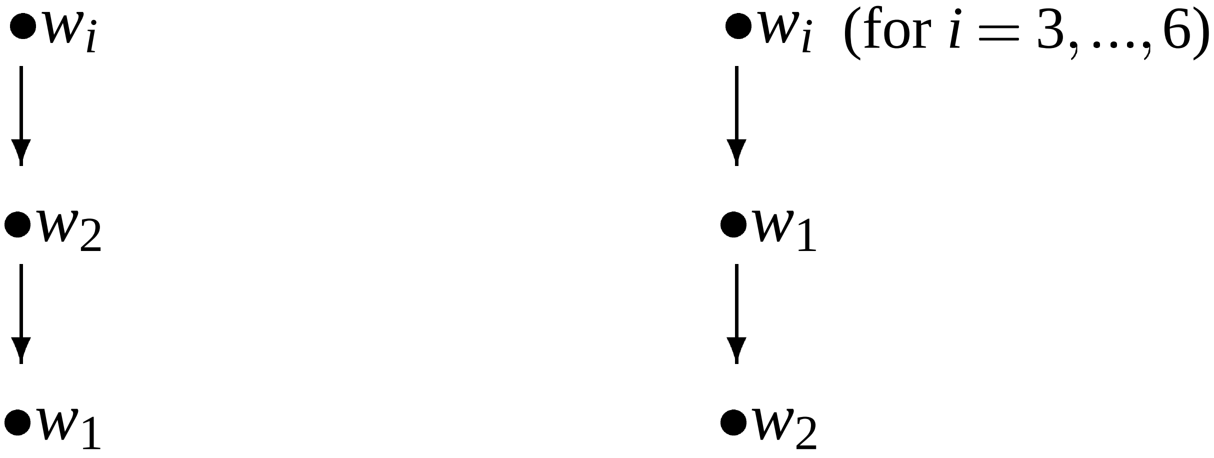
$$w_2 \preceq' w_1 \text{ iff } w_2 \in \min_{\mathcal{M}}(A) \quad \text{(b)}$$

$(\mathscr{P}_2)$ If $w_1, w_2 \notin \min_{\mathcal{M}}(A)$ then: $w_1 \preceq' w_2$ iff $w_1 \preceq w_2$.

Intuitively: the ordering is left unaltered except as indispensably required by the fact that the set of minimal A-worlds must now form the set of "newly" minimal worlds.

Commutation

Model $\mathcal{M}$ Model $\mathcal{M}^{\star}o$



Before the offence At the time of the offence

$\mathcal{M} \models \bigcirc \neg r_1$ $\mathcal{M}^{\star}o \models \bigcirc r_1$

Problem: we also have $\mathcal{M}_1^{\star}o \models \bigcirc o$, viz. at the time of the transgression, the primary obligation is no longer is force

- Already a problem for the temporal approach:
  $A \rightarrow [S]A \rightarrow \bigcirc A$ ([S]=settledness)

- Harmonize the two treatments by requiring the input to be true in every possible worlds $\mathcal{M}_1^{\star}o \models \bigcirc o$ just means that o is vacuously obligatory.

- We would like to be able to say that o should not have been done even though it is settled as true

The full sequence

One-move interchange -

Input	Induced ordering		Perms	Output
o	$\mathcal{M}_1^\star o$	$w_2 \prec w_1 \prec w_{i(i\geq 3)}$	1	$\bigcirc r_1$
$\neg o$	$\mathcal{M}_1^\star \neg o$	$w_1 \prec w_2 \prec w_{i(i\geq 3)}$	0	$\bigcirc \neg r_1$

Two-move interchange -

Input sequence	Induced ordering		Perms	Output
1. o, r_1	$(\mathcal{M}_1^\star o)^\star r_1$	$w_2 \prec w_1 \prec w_i$	1	$\bigcirc r_2$
2. $o, \neg r_1$	$(\mathcal{M}_1^\star o)^\star \neg r_1$	$w_1 \prec w_2 \prec w_i$	2	$\bigcirc \neg r_2$
3. $\neg o, r_1$	$(\mathcal{M}_1^\star \neg o)^\star r_1$	$w_2 \prec w_1 \prec w_i$	1	$\bigcirc r_2$
4. $\neg o, \neg r_1$	$(\mathcal{M}_1^\star \neg o)^\star \neg r_1$	$w_1 \prec w_2 \prec w_i$	0	$\bigcirc \neg r_2$

Three-move interchange -

Input sequence	Induced ordering		Perm	Output
1. o, r_1, r_2	$((\mathcal{M}_1^\star o)^\star r_1)^\star r_2$	$w_2 \prec w_1 \prec w_i$	1	$\bigcirc a$
2. $o, r_1, \neg r_2$	$((\mathcal{M}_1^\star o)^\star r_1)^\star \neg r_2$	$w_1 \prec w_2 \prec w_i$	2	$\bigcirc \neg a$
3. $o, \neg r_1, r_2$	$((\mathcal{M}_1^\star o)^\star \neg r_1)^\star r_2$	$w_2 \prec w_1 \prec w_i$	3	$\bigcirc a$
4. $o, \neg r_1, \neg r_2$	$((\mathcal{M}_1^\star o)^\star \neg r_1)^\star \neg r_2$	$w_1 \prec w_2 \prec w_i$	2	$\bigcirc \neg a$
5. $\neg o, r_1, r_2$	$((\mathcal{M}_1^\star \neg o)^\star r_1)^\star r_2$	$w_2 \prec w_1 \prec w_i$	1	$\bigcirc a$
6. $\neg o, r_1, \neg r_2$	$((\mathcal{M}_1^\star \neg o)^\star r_1)^\star \neg r_2$	$w_1 \prec w_2 \prec w_i$	2	$\bigcirc \neg a$
7. $\neg o, \neg r_1, r_2$	$((\mathcal{M}_1^\star \neg o)^\star \neg r_1)^\star r_2$	$w_2 \prec w_1 \prec w_i$	2	$\bigcirc a$
8. $\neg o, \neg r_1, \neg r_2$	$((\mathcal{M}_1^\star \neg o)^\star \neg r_1)^\star \neg r_2$	$w_1 \prec w_2 \prec w_i$	0	$\bigcirc \neg a$

After the 1st move, there is a permutation each time the ‘polarity’ of the input sentence changes

Before the offence has been anticipated

Premisses

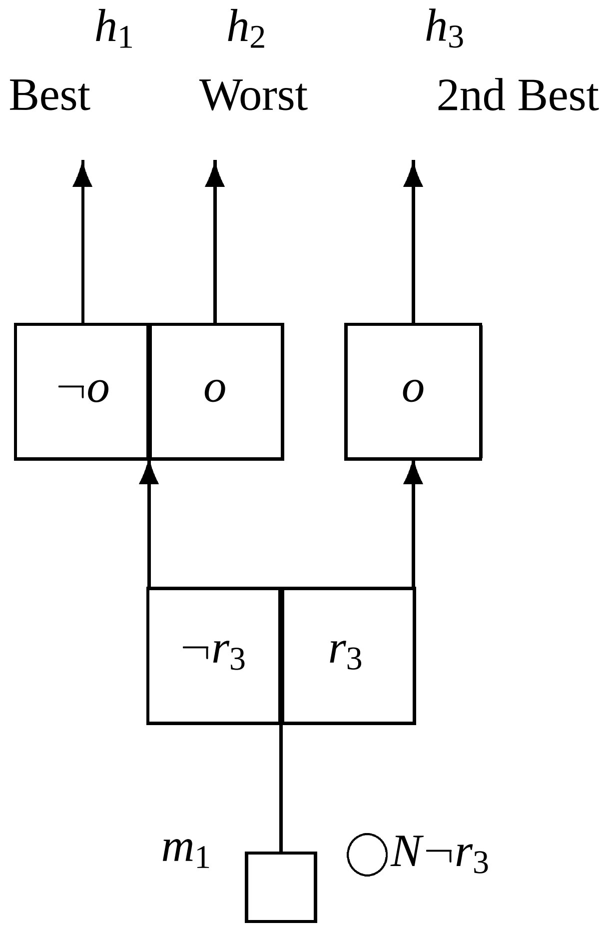
$\bigcirc NN\neg o$

$\bigcirc(Nr_3/NNo)$

$\Box(Nr_3 \rightarrow NNo)$

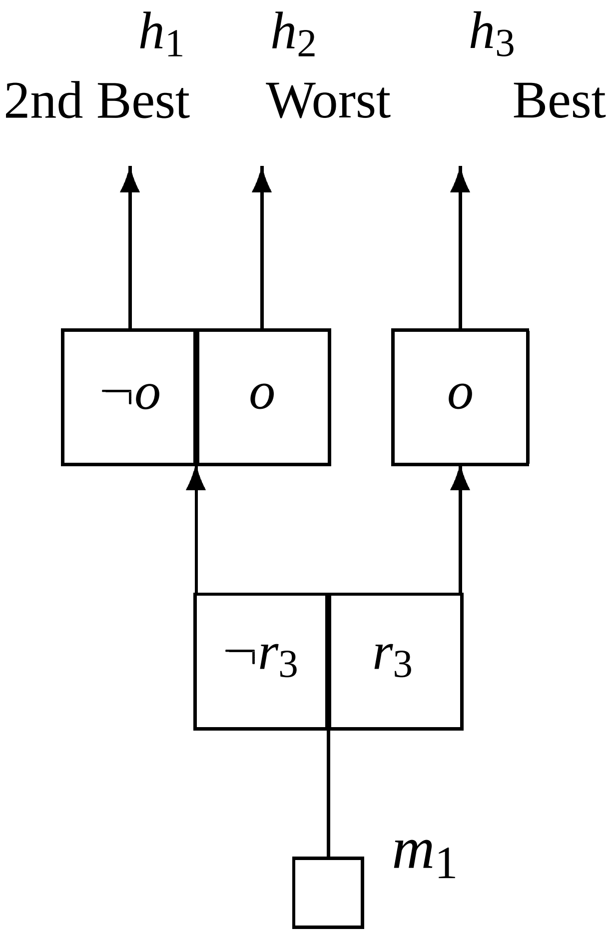
In model $\mathfrak{M}_1$:

$$\mathfrak{M}_1, m_1/h_1 \models \bigcirc N\neg r_3$$



Once the offence has been anticipated

$\mathfrak{M}^{\circledast^m}\phi$ = the model obtained from $\mathfrak{M}$ once the occurrence of ϕ has been anticipated at moment m.
The relation $\preceq'$ in $\mathfrak{M}^{\circledast^m}\phi$ is defined along the same lines as in the timeless framework.



In model $\mathfrak{M}_1^{\circledast^{m_1}} NNo$:
$\mathfrak{M}_1^{\circledast^{m_1}} NNo, m_1/h_1 \models \bigcirc N r_3$

For future research

- At the time of the offence, the primary obligation $\bigcirc\neg o$ is no longer in force. How can this be avoided?
- What does the construction have to say about the other kinds of Ctd structures discussed in the literature?
- What about the other requirements that a formalization of Ctd scenarios can reasonably be expected to meet?