The Effect of Spring Stiffness and Control Gain with an Elastic Rate Control Pointing Device

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ABSTRACT
Isometric and elastic devices are most compatible with a rate control mapping. However, the effect of elastic stiffness has not been thoroughly investigated nor its interaction with control gain. In a controlled experiment, these factors are investigated along with user feedback regarding ease-of-use and fatigue. The results reveal a U-shaped profile of control gain vs. movement time, with different profiles for different stiffness levels. Using the optimum control gain for each stiffness level, performance across stiffness levels was similar. However, users preferred lower stiffness and lower control gain levels due to increased controller displacement. Based on these results, design guidelines for elastic rate control devices are given.

ACM Classification: H5.2 [Information interfaces and presentation]: User Interfaces. - Graphical user interfaces.

General terms: Human factors

Keywords: elastic, control gain, stiffness, rate control

INTRODUCTION
The mouse is an efficient pointing device [4,13,14], but there are environments without a flat surface where the mouse is not practical. Laptop manufacturers have responded with alternative pointing devices such as the touchpad. Like the mouse, the touch pad is an isometric input device (it is free-moving and uses X-Y position as input) with a position control mapping (the input is mapped to an X-Y cursor position) [21]. However, the touch pad has a very small input area and requires frequent clutching which degrades performance [5]. Clutching can be reduced by increasing the ratio of control movement to display movement (Control-Display gain, or CD gain) [1,3,9,10], but very high CD gain levels can hurt performance [1,9,10]. Alternatively, clutching can be removed altogether by using a rate control mapping where the device input is mapped to a cursor velocity and direction. A rate control mapping is more suitable for an isometric or elastic device since they have a self-centering mechanism to return the device to a neutral state when released [21]. Isometric devices, such as the TrackPoint [15,17], do not perceptibly move and instead measure the force applied. Unfortunately, isometric devices seem to be faster than isometric devices [6,7,14,16]. However, this difference could be due to non-optimal device parameters. For example, isometric devices are also affected by control gain [9,11] and after some informal parameter tuning, Zhai found no difference between isometric and isometric 6 DOF devices [21]. Another issue is that isometric devices lack proprioception, the human sense of position and movement of limbs, and may increase fatigue [21]. In contrast, elastic devices have an effector which can be displaced over a certain operating range, with a spring applying an opposite force to self-centre (Figure 1). Yet, with the exception of Zhai’s small pilot experiment with a 6 DOF input device [21], little is known about the effect of elastic device spring stiffness and there is no clear conclusion for the added influence of control gain [4,7,9,11,18]. This, in spite of elastic devices appearing in the literature [5,8,12]. Without an understanding of the combined effect of elastic stiffness and control gain, tuning parameters for isometric or elastic devices will continue to be ad hoc. In this paper we present an experiment that systematically evaluates the interaction between control gain and stiffness using a high performance force feedback device. We found that the control gain vs. movement time has a U-shaped profile and in addition, that proprioception influences the shape of the U: with a carefully chosen control gain, elastic and isometric devices can perform equally well. However, our participants preferred more elasticity. We also show that operating range is not only affected by stiffness, but also by control gain. Finally, using these results, we give guidelines for the design and use of elastic and isometric rate control devices given the stiffness and operating range.

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Figure 1. Elastic device composed of a spring attached to an effector. The resistive force is proportional to the effector displacement which is limited by the operating range.
BACKGROUND AND RELATED WORK
Elastic device stiffness
An elastic device is composed of an effector and spring placed in a constrained movement area, called the operating range (Figure 1). The user pushes on the effector and the device measures either the distance from the effector to the neutral position (the displacement) or the applied force, and uses this as input.

\[ F = -k \cdot d \]  

(1)

Using Hooke's law [20], the reactive force \( F \) is a linear function of spring stiffness \( k \) and displacement \( d \) (Equation 1). Spring stiffness is measured in newtons per metre (N.m\(^{-1}\)). To give some idea of the range of stiffness, a typical office stationary rubber band is between 30 N.m\(^{-1}\) and 300 N.m\(^{-1}\). With high spring stiffness, the effective range of displacement decreases and force is a more appropriate measurement. In the extreme case, when stiffness is infinite with no displacement, the behavior is that of an isometric device. With lower spring stiffness, the effective range of applied force decreases and the effector distance is the more appropriate measurement. When stiffness is set to zero, the behavior is that of an isotonic device.

Effect of stiffness
To our knowledge, only Zhai has studied the influence of stiffness on user performance, but only in a pilot study with 2 users [21]. He used a custom 6 DOF built device that could vary the stiffness by changing the number of elastic bands. Although the range of stiffness evaluated is not reported he found optimal performance around 120 N.m\(^{-1}\).

In a follow-up study, Zhai compared a 6 DOF isometric device with a 6 DOF elastic device (120 N.m\(^{-1}\) – 20 mm operating range) in a 6 DOF docking task [21]. He did not find a significant difference in performance, but found differences in user fatigue. Unlike an isometric device, an elastic device can be displaced to provide proprioception – our sense of the joint positions in our body. Zhai summarizes the tradeoff with elastic stiffness settings: “For the sake of compatibility with rate control, a stiff (therefore strongly self-centered) elasticity is desirable. However, a stiff (near isometric) elastic device provides less rich proprioception than a loose one that allows more movement within a range of non-fatiguing forces.” [21] In spite of Zhai’s argument there are no design guidelines (aside from his initial pilot study results) for selecting an optimum stiffness value which balances strong self-centering and rich proprioception.

Control gain
With an isotonic position control device such as a mouse, CD gain is a unit free ratio since both input and output are corresponding movements. For example, with a CD gain of 2, the display pointer moves twice as far as the corresponding movement of the control device.

However, with isometric or isotonic rate control devices, input and outputs are expressed in different units. With an isometric device, the applied force \( F \), expressed in N, is mapped to a pointer velocity \( V \), expressed in m.s\(^{-1}\). Thus, the isometric control gain (\( CG_{\text{isometric}} \)) is expressed in m.s\(^{-1}\).N\(^{-1}\) and the mapping function is:

\[ V = CG_{\text{isometric}} \cdot F \]  

(2)

As a simple illustration, given a control gain of 2 m.s\(^{-1}\).N\(^{-1}\), the same force on the isometric controller will produce twice the velocity of the cursor relative to a control gain of 1 m.s\(^{-1}\).N\(^{-1}\) [11].

With a pure isotonic rate control device, the displacement distance \( d \), expressed in meters, is mapped to a pointer velocity \( V \). Here, the isotonic control gain (\( CG_{\text{isotonic}} \)) is expressed in s\(^{-1}\) and the mapping function is:

\[ V = CG_{\text{isotonic}} \cdot d \]  

(3)

With an elastic device, as the pointer velocity can be computed from either displacement or applied force, either Equation 2 or 3 can be used. Moving from one mapping function to the other is simply done by dividing or multiplying the control gain by the spring stiffness, since the resistive force is proportional to displacement (Equation 1):

\[ CG_{\text{isotonic}} = k \cdot CG_{\text{isometric}} \]  

(4)

Effect of control gain
Constant control gain transfer functions
Gibbs [9] tested 7 values of control gain with an elastic rate control device and found that performance follows a U-shape, described by the following equation:

\[ T = 0.949 + 0.255 \cdot CG_{\text{isotonic}} + \frac{0.444}{CG_{\text{isotonic}}} \]  

(5)

This gives an optimal performance at approximately 1.3 s\(^{-1}\), and then degrades with higher or lower levels. He used a “slightly loaded” joystick with a 100 mm operating range (±25° of angular movement and 120 mm joystick length). For comparison sake, we estimated “slightly loaded” to be similar to the elasticity of a loose rubber band and assigned a stiffness of 30 N.m\(^{-1}\). With this, we can estimate the control gain range, using Equation 4, to be 0.01 – 0.1 m.s\(^{-1}\).N\(^{-1}\). The stimulus was a 3 mm target at a distance of 22.5 mm (with an index of difficulty, or ID of 3). Error rate was not a reported factor since each pointing task had to be completed successfully.

Using an isometric rate control device, Kantowitz and Elvers [11] tested two levels of control gain (0.04 and 0.08 m.s\(^{-1}\).N\(^{-1}\)) and found no significant difference. Error rates were between 15 and 30%. Interestingly, their study also evaluated an isometric position control device with control gain levels of 1 and 2. They found that Fitts’ law held for both devices, with higher slopes for the rate control condition and negative intercepts close to 0.8s. Using an isotonic rate control device, Epps [7] also found good regression fitness with a negative intercept equal to 0.587 s.
Non-Linear Control Gain Transfer Functions

Researchers have also experimented with non-linear transfer functions for control gain.

Card et al. [4] tested an isometric joystick with a simple non-linear parabolic transfer function. The transfer function mapped forces below 4 N to a control gain of 0 m.s\(^{-1}\).N\(^{-1}\) (the cursor did not move) and 0.008 m.s\(^{-1}\).N\(^{-1}\) for forces above. In spite of good Fitts’ law regression fitness, the joystick had an error rate between 10 and 15% and lower performance than the mouse. The authors attributed this to the non-linear transfer function.

Rutledge et al. [17,18] compared a custom non-linear transfer function with three constant control gain settings (0.075, 0.15, 0.3 m.s\(^{-1}\).N\(^{-1}\)) and 2 parabolic non-linear functions for an isometric rate control device. For all functions, performance decreased with control gain 0.3. Also, the constant functions were found to be faster when the index of difficulty is below 3.5 bits and slower above. Results for all functions were found to follow Fitts’ law. Their function has been improved by adding a negative inertia filter to the pointer motion [2].

Zhai ran a small pilot experiment with two subjects (an expert and a novice) to tune the control gain of an elastic device and an isometric device [21]. He used the non-linear function: \( V = F^{CG} \). Like Gibbs, he found that performance was optimal with a mid-level control gain, but since each level was tested with only 12 trials, it is unclear if his findings are statistically significant. Zhai characterized this as a “U-Shape,” based on a plot of mean selection times over a range of control gains. However, he does not report the maximum force and the units of control gain are unclear.

### Summary

There is no clear conclusion for the effect of control gain on user performance for rate control devices (Table 1). Gibbs [9] and Zhai [21] found a U-shaped performance curve; Rutledge et al. [18] found performance degraded with high control gains; and Kantowitz and Elvers [11] did not find any effect. However, in these experiments, the range of control gain levels was too small or the target distances and widths were conservative. Only Rutledge et al. [18] compared non-linear and constant control gain transfer functions, and they found that performance decreased with high control gain. In addition, only Zhai’s 2-person pilot study has examined the influence of stiffness on user performance [21]. Yet, in his follow up evaluation of elastic and isometric devices, no significant effect on performance was found. This, in spite of participants noting increased fatigue with the isometric device and Zhai’s argument that the elastic device provides richer proprioception. Finally, the interaction between control gain and stiffness has not been evaluated.

### EXPERIMENT

We wished to investigate the effect of control gain and stiffness for elastic devices ranging from isotonic to nearly isometric. Rather than confound the experiment with a custom non-linear control gain transfer function, we focus only on constant transfer functions. If there is an effect for control gain, then researchers can design non-linear transfer functions guided by these base line results.

### Apparatus

We used a single Phantom Desktop haptic device to simulate the different elastic devices [19]. This eliminated extraneous intra-device differences, such as ergonomics, size and sensitivity, while also providing an efficient way to administer the experiment without having to frequently swap custom-built elastic devices during a session.

The Phantom uses a stylus connected to a force-feedback armature to produce haptic feedback. It has an 1100 DPI nominal resolution and an operating range of 160 mm. With a maximum resistive force of 7.9 N, the Phantom can simulate a maximum stiffness of 1860 N.m\(^{-1}\). This allowed us to simulate a large stiffness range, from isotonic to near isometric (the maximum stiffness felt like an inflated bicycle tire). Using the force feedback, the movement of the

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### Table 1: Summary of prior work evaluating the effect of control gain with rate control devices.

(*control gain levels estimated using 30 N.m\(^{-1}\) stiffness; **width between 1 and 10 characters with 1 character = 2.46 mm [13])

<table>
<thead>
<tr>
<th>Study</th>
<th>Device</th>
<th>Control gain</th>
<th>ID</th>
<th>Width (mm)</th>
<th>Dist (mm)</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gibbs [9]</td>
<td>Elastic joystick</td>
<td>Constant Function</td>
<td>3</td>
<td>3</td>
<td>22.5</td>
<td>U-Shape: best control gain at 1.3 s(^{-1}).0.03 m.s(^{-1}).N(^{-1}))</td>
</tr>
<tr>
<td>Kantowitz &amp; Elvers [11]</td>
<td>Isometric joystick</td>
<td>Constant Function (0.04, and 0.08 m.s(^{-1}).N(^{-1}))</td>
<td>3.5 - 5.5</td>
<td>7 - 11</td>
<td>63 - 168</td>
<td>No effect</td>
</tr>
<tr>
<td>Card et al. [4]</td>
<td>Isometric joystick</td>
<td>Non-Linear Function (0 to 0.008 m.s(^{-1}).N(^{-1}))</td>
<td>0.5 - 6</td>
<td>2.46 - 24.6**</td>
<td>10 - 160</td>
<td>High Error Rates</td>
</tr>
<tr>
<td>Rutledge et al. [18]</td>
<td>Isometric pointing stick</td>
<td>3 Non-Linear and 3 Constant Functions (0.075, 0.15, 0.3 m.s(^{-1}).N(^{-1}))</td>
<td>1.75 - 6</td>
<td>2.8 – 14</td>
<td>Random</td>
<td>Negative Effect for High control gain: performance decreased with control gain 0.3</td>
</tr>
<tr>
<td>Zhai [21]</td>
<td>Elastic device</td>
<td>Non-Linear Function (control gain values unclear)</td>
<td>6 DOF Docking Task</td>
<td>U-Shape</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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effector was constrained to a single degree of freedom parallel to the display and 2 cm above the table. To avoid instability when selecting the target, participants used their non-dominant hand to trigger a button on a mouse.

The pointer velocity was computed from the resistive force and control gain using Equation 2. After running a pilot experiment, we observed that a dead-band was needed to prevent the pointer from moving without any apparent force applied to the device. The problem is that the Phantom will not fully self-centre due to the presence of backdrive friction. We found that using a deadband force, $F_{DB}$, of 0.055 N stabilized self-centering across all stiffness levels. This value is close to the Phantom’s rated backdrive friction of 0.06 N and similar to values used previously [4,18]. Including the deadband, the pointer velocity is computed as:

$$V = \begin{cases} 0 & \text{if } |F| < F_{DB} \\ \text{Sgn}(F) \cdot CD \cdot (|F| - F_{DB}) & \text{otherwise.} \end{cases} \quad (6)$$

Where $\text{Sgn}(x) = -1$ if $x < 1$, 1 if $x > 1$, 0 if $x = 0$.

The simulation is coded in C++ and uses OpenGL for the display and OpenHaptics toolkit to control the Phantom. The frequencies of the visual and haptic renderings were 60 Hz and 1000 Hz respectively.

**Task**

The task was a reciprocal one dimensional pointing task (Figure 2). Each experimental trial began after the previous target was successfully selected and ended with the selection of the current target. After the current target was successfully selected, it turned grey, and the next target to be selected, on the other side of the screen, turned white. If a participant missed a target, a sound was heard and an error was logged. Participants had to successfully select the current target before moving to the next, even if it required multiple clicks. The pointer was not constrained to the bounds of the screen to avoid using the edges to assist in target acquisition.

**Participants**

Sixteen right-handed people (14 male, 2 female) with a mean age of 25.2 (SD = 2.9) participated. All were regular computer users (at least 2 hours per day) and had little or no experience with isometric devices such as the TrackPoint or SpaceMouse, nor did they have previous experience with the Phantom.

**Design**

A repeated measures within-subjects design was used. The independent variables were STIFFNESS (0, 30, 120, 800, 1600 N.m$^{-1}$), Control Gain (CG) (0.05, 0.1, 0.3, 0.5, 0.7 m.s$^{-1}$ N$^{-1}$), target DISTANCE ($D_L = 300$ mm, $D_M = 150$ mm, $D_S = 75$ mm), and target WIDTH ($W_L = 8$ mm, $W_M = 4$ mm, $W_S = 2$ mm). The operating range is treated as a dependant variable since it is a function of CG and STIFFNESS.

The nine DISTANCE and WIDTH (D-W) combinations give five Fitts’ Indices of difficulty (ID) ranging from 3.4 to 7.2. This design gives us redundant points at each ID, allowing us to examine different effects of DISTANCE and WIDTH. The tradeoff is that our ID range of 3.8 is narrower than Card et al. [4] or Epps [7] (with 6.18 and 6.04 respectively). Mackenzie [13] discusses this issue, noting extremely narrow ID ranges, such as 2, as problematic. Our mid-point choice is an attempt to balance these range breadth with redundant points for each ID.

The STIFFNESS levels range from pure isotonic (0 N.m$^{-1}$) to nearly pure isometric (1600 N.m$^{-1}$ with an operating range less than 1.0 mm). The intermediate STIFFNESS levels of 120 N.m$^{-1}$ and 30 N.m$^{-1}$ enable comparison with the results of Zhai and Gibbs respectively. We also added a stiff spring of 800 N.m$^{-1}$. The CG levels were chosen after running a pilot study with two participants. Since there is no resistive feedback with a stiffness of 0 N.m$^{-1}$, we used a 30 N.m$^{-1}$ stiffness to compute the velocity making the only difference between these two levels the resistive force. Note that the SpaceMouse Cadman has a stiffness of approximately 2500 N.m$^{-1}$ and an operating range of 4mm.

Participants completed two consecutive BLOCKS of trials for each combination of CG and STIFFNESS. Each BLOCK consisted of 27 trials: 3 repetitions of the 9 D-W combinations. The D-W combinations were presented in ascending order of ID within a single BLOCK to avoid drastic changes in difficulty. After 3 trials, a message displayed the cumulative error rate and encouraged participants to conform to a 4% error rate by speeding up or slowing down. The presentation order of CG and STIFFNESS was counterbalanced from low-to-high and high-to-low across participants using a Latin Square design.

Before starting the experiment, participants had a 5-10 minute training period to get used to controlling the cursor with 120 N.m$^{-3}$ STIFFNESS and CG of 0.1 m.s$^{-1}$ N$^{-1}$. After the two blocks were completed for each STIFFNESS and CG combination, the participant rated the settings ease-of-use.
and fatigue using a 5 point Likert scale. The experiment lasted approximately 120 minutes.

In summary, the experimental design was:
- 16 participants ×
- 5 STIFFNESS × 5 CG ×
- 2 BLOCKS × 9 D-W combinations × 3 trials
  = 21,600 total trials

RESULTS
The dependent variables were error rate, movement time, operating range, and maximum force.

Performance

Error rate
Repeated measures analyses of variance showed that the order of presentation of STIFFNESS or CG had no significant effect or interaction on error rate, indicating that a within-participants design was appropriate. We also found no significant effect or interaction for BLOCK indicating there was no presence of a learning effect. Repeated measures analysis of variance found an expected significant main effect for WIDTH (F_{2,30} = 48.5, p < 0.0001) on error rate, but also significant main effects for STIFFNESS (F_{4,60} = 10.5, p < 0.0001) and CG (F_{4,60} = 25.1, p < 0.0001).

However, we have to be cautious before drawing any conclusions regarding these main effects because of a significant interaction between STIFFNESS and CG (F_{16,240} = 4.3, p < 0.0001) which shows that error rate increases at different rates according to CG and STIFFNESS (Figure 3) – no other significant interaction was found. For example, while participants maintained an error rate between 2.8 and 4.7% for 30 N.m⁻¹ STIFFNESS, they failed to do so with a STIFFNESS of 1600 N.m⁻¹ where the error rate ranged from 2.6% to 16.5%. This is in spite of encouraging participants to try and maintain a 4% error rate for all settings (note that Kantowitz et al. [11] and Card et al. [4] also found error rates between 10 and 15%).

Given this STIFFNESS and CG interaction, a more meaningful comparison of error rate across STIFFNESS levels should use optimum levels of CG. We first define the optimum CG range for each STIFFNESS level as the range of CG levels with a statistically significant lower error rate. In other words, the error rates for each CG within the optimum range are statistically better than those for all CG levels outside the range, and the error rates for CG levels within the range have no statistical difference. Using pairwise comparisons, we found the following optimum ranges:

- STIFFNESS 0: CG [0.05 – 0.5] (all p < 0.02)
- STIFFNESS 30: CG [0.05 – 0.3] (all p < 0.044)
- STIFFNESS 120: CG [0.05 – 0.1] (all p < 0.014)
- STIFFNESS 800: CG [0.05 – 0.1] (all p < 0.022)
- STIFFNESS 1600: CG [0.05 – 0.1] (all p < 0.004)

Repeated measures analysis of variance using the optimum CG range found a significant main effect (F_{4,60} = 4.1, p = 0.006) for STIFFNESS on error rate. Pairwise comparisons found that STIFFNESS 0 had a significantly lower error rate of 5% compared to STIFFNESS 30, 120, and 1600 (p < 0.007) (Figure 4).

Our results indicate that CG has an impact on the error rate, and by comparing the optimal CG ranges, we found a slight error rate advantage with a STIFFNESS of 30 N.m⁻¹. However, we note that all error rates are within an acceptable range at close to 4%.

Movement time
Movement time is defined as the time it took to move from the previous target to the current target and correctly select it. Targets that were not selected on the first attempt were marked as errors, but were still included in the timing analysis (the same significant effects were found with and without errors).

Repeated measures analyses of variance showed that the order of presentation of STIFFNESS or CG had no significant effect or interaction on movement time, indicating that a within-participants design was appropriate. We also found no significant effect or interaction for BLOCK indicating there was no presence of a learning effect. As predicted by
Fitts’ law, repeated measures analysis of variance found significant effects for distance \((F_{2,16} = 280, p < 0.0001)\) and width \((F_{2,16} = 190, p < 0.0001)\) on movement time, but also main effects for stiffness \((F_{4,32} = 3.6, p = 0.016)\) and CG \((F_{4,32} = 6.9, p < 0.0001)\). Comparisons found a significant difference between stiffness 0 and all other stiffness values \((p < 0.02)\). At 3.8s, the isotonic equivalent stiffness value of 0 was 15% slower than the elastic or isometric stiffness values. This confirms the results found by Zhai [21] while extending it to a wider range of stiffness values. However, Zhai found a 47% difference between the isotonic and isometric rate control devices, which may be explained by the 6 DOF docking task or the different device form factors he used. Perhaps most interesting is that our results suggest that even with a very low stiffness of 30 N.m\(^{-1}\), participants can take advantage of the resistive feedback to more accurately control the device.

**Fitts’ Law analysis**

We also found a significant stiffness × CG × ID interaction on movement time \((F_{64,768} = 3.5, p < 0.01)\) which led us to a Fitts’ law analysis. To perform the Fitts law analysis, we computed the effective width for each stiffness × CG × D-W combination, according to MacKenzie’s formulation since the error rate is not constant [13]. By using the nine D-W combinations rather than an aggregate time for the five IDs, independent effects of distance and width may be exposed. We found negative intercepts between -1.75 s and -0.11 s and slopes between 0.64 and 1.0 s.bit\(^{-1}\) for those settings which followed Fitts’ law. These are close to those found in previous work [9,7].

Fitts’ law does not hold for all settings (Table 2 & Figure 8). The settings which do not hold are with the largest distance and low CG levels, or the smallest width and high CG levels. These results explain the stiffness × CG × ID interaction.

**Operating range**

The operating range is defined as the maximum distance between the two extreme positions of the effector. We expected the main effect of stiffness on operating range \((F_{4,32} = 173.5, p < 0.0001)\) since there is a dependency, but we were more interested in how CG and distance affected operating range. Repeated measures analysis of variance found a significant main effect for distance \((F_{3,16} = 529.6, p < 0.0001)\) with operating range increasing with distance. Pairwise comparisons found significant differences between all distance values \((p < 0.0001)\).
movement is required for devi
tate control devices. For example, very little physical
These operating range curves can assist designers of elastic
small (less than 1mm in some cases).

We found a main effect for CG (F(4,32) = 137.5, p < 0.0001), but again the significant interactions are more relevant. We found a STIFFNESS × CG interaction (F(16,128) = 50.8, p < 0.0001) which shows how the operating range decreases with increased CG but at different rates across STIFFNESS (Figure 7). For example, the operating range decreases from 154 mm to 32 mm for STIFFNESS 0 but decreases from 3.5 mm to 0.7 mm for STIFFNESS 1600 N.m⁻¹. A significant CG × STIFFNESS × DISTANCE interaction (F(12,250)=10.8, p < 0.0001) revealed that with low STIFFNESS and low CG, the operating ranges for different DISTANCE can vary by as much as 40 mm. But, as the STIFFNESS and CG increase, the difference across DISTANCE becomes very small (less than 1mm in some cases).

These operating range curves can assist designers of elastic rate control devices. For example, very little physical movement is required for devices with stiffness above 800 N.m⁻¹ with most distances. In addition, the relationship between these three factors can be further formalized using a multiple regression analysis.

Using a regression analysis with a variety of possible model predictors, we found the ratio of distance to stiffness D/k and the inverse of the product of stiffness and control gain 1/(k · CG) to be significant predictors (p<0.0001). A forward stepwise analysis found the contribution of each predictor to be:

\[ OR = 2 + \frac{5 \cdot D}{k} + \frac{146}{k \cdot CG} \]  

(7)

With this model, D/k explains 83.7% of the variance (std err 5.2) and D/k 13.8% (std err 0.28). With a significant r² value of 0.975 (F(2,59)=1114, p < 0.0001) and no within-predictor collinearity (VIF=1.37, tolerance 0.73), this model predicts the operating ranges used by our experimental settings within -80% and 41% of the real data, with a mean prediction within -15%.

**Maximum force**

The maximum force is the maximum absolute amplitude of force applied to the end effector during a trial. Repeated measures analysis of variance show a significant main effect of STIFFNESS (F(4,32) = 57.0, p < 0.0001), CG (F(4,32) = 84.2, p < 0.0001), and DISTANCE (F(2,16) = 58.4, p < 0.0001) on maximum force. But again, the significant STIFFNESS × CG interaction is most interesting (F(16,128) = 17.2, p < 0.0001) (Figure 9). It shows that the maximum force applied on the effector decreases at different rates across stiffness as the CG increases. For example, it decreases

![Figure 8. Fitts' law regression plots by STIFFNESS and CG. Plots highlighted in bold have low fitness (r² < 0.8). In these cases, different D-W combinations have different movement times in spite of having the same ID. For example, the combinations D₁,W₁, D₀,W₀, and D₀,W₁ all have ID = 5.3, but for STIFFNESS and CG settings of 1600 N.m⁻¹ and 0.7 m.s⁻¹.N⁻¹ the movement times appear different.](image-url)

![Table 2: Fitts' Law regression values for STIFFNESS and CG: a is the intercept, b is the slope, r² is the fitness (computed as means over participants). Lines highlighted in grey have low fitness (r² < 0.8).](table-url)
from 2.8 N to 0.6 N for the 1600 N.m$^{-1}$ STIFFNESS and from 1.6 N to 0.5 N for the 30 N.m$^{-1}$ STIFFNESS.

We were pleased to see that participants did not exceed the maximum force of the Phantom (7.9 N), and in fact applied forces much less than that. These values can help elastic device designers when choosing appropriate force sensing hardware.

User feedback of ease-of-use and fatigue

After completing all BLOCKS for a STIFFNESS and CG combination, we asked the participants to answer two questions about ease-of-use and fatigue using a Likert scale. The ease-of-use question was worded “How easy is it to move the pointer the way you want?” and the fatigue question “How tiring is it to move the pointer?” The Likert scale ranged from 1 (“very easy to use” or “not tiring at all”) to 5 (“very difficult to use” or “very tiring”). Since this produces ordinal data which is not normally distributed, the analysis used rank-transformed data – replacing each original value by its rank from 1 for the smallest value to N for the largest. Note that there is no equivalent non-parametric method for two-way or three-way designs.

Ease-of-use

Repeated measures analysis of variance found a significant main effect for STIFFNESS ($F_{4,60} = 13.3, p < 0.0001$) and CG ($F_{4,60} = 12.9, p < 0.0001$) on ease-of-use scale. Again, a significant STIFFNESS $\times$ CG interaction ($F_{16,240} = 4.9, p = 0.0001$) is most relevant (Figure 10). It shows that the easiest to use control gain depends on stiffness. For low values of STIFFNESS, pairwise comparisons found no significant difference across CG. However, as the STIFFNESS increases, significant differences were found for a stiffness 120 N.m$^{-1}$ with a significant difference between CG 0.7 m.s$^{-1}.N^{-1}$ and all other CG levels ($p < 0.015$) and for STIFFNESS 800 N.m$^{-1}$ and 1600 N.m$^{-1}$, CG ranges [0.05-0.1] and [0.3-0.7] were significantly different ($p < 0.03$).

If we compare the optimum CG ranges for each STIFFNESS, we see a significant difference between 0 N.m$^{-1}$ and all other values except 1600 N.m$^{-1}$ ($p < 0.01$) (Figure 11).

Fatigue

Repeated measures analysis of variance found a significant main effect across STIFFNESS ($F_{4,60} = 7.8, p < 0.001$) and CG ($F_{4,60} = 10.4, p = 0.001$) on fatigue scale. Once again, a significant STIFFNESS $\times$ CG interaction ($F_{16,240} = 2.6, p = 0.001$) is most relevant. Pairwise comparisons found no significant differences for CG with 0 N.m$^{-1}$ STIFFNESS. However, for higher STIFFNESS levels, participants found either the lowest or highest extremes of CG to be most tiring (Figure 12). For STIFFNESS 30 N.m$^{-1}$, the lower range of CG [0.05 - 0.1] were most tiring ($p < 0.0001$); for STIFFNESS 120 N.m$^{-1}$, the lower range [0.05-0.1] were most tiring ($p < 0.03$); but for STIFFNESS 800 N.m$^{-1}$, the high range [0.1 – 0.5] were least tiring ($p < 0.02$); for STIFFNESS 1600 N.m$^{-1}$ there was no significant difference.

By using the optimum CG ranges reported as least tiring, we find significant differences between STIFFNESS ranges [30 – 120 N.m$^{-1}$] and [800 – 1600 N.m$^{-1}$] ($p < 0.02$) (Figure 11). Participants found higher STIFFNESS more tiring confirming Zhai’s argument that although performance may be similar, people find isometric feedback more tiring.
Influence of stiffness

With an appropriate selection of control gain, devices with different stiffness perform equally well, except for stiffness 0, the pure isotonic condition, which had slightly higher error rates and a 15% increase in movement time. The only difference between 0 and 30 N.m\(^{-1}\) stiffness levels is the self-centering mechanism, so our study confirms that an elastic or isometric device is more suited to rate control. But, we also demonstrate that even a moderate level of stiffness is sufficient. The difference in performance between the pure isotonic and the elastic/isometric condition is lower than Zhai [10] likely because we use a 1D controller and a single device form factor across conditions.

Design guidelines

Considering the importance of proprioception, fatigue and the difficulty of tuning control gain with high stiffness, we recommend that elastic device designers use lower stiffness values. However, with lower stiffness values comes a larger device operating range. This can be an issue because rate control devices are often used for situations where the physical device space is restricted. If this is an issue, then we recommend using the upper limit of the optimum range of control gain values, and continue to use a stiffness value as low as possible to obtain the desirable operating range.

Before setting the control gain, first select a deadband force to counteract the natural backdrive friction of the device. Also, to accurately control the cursor velocity, the display resolution and control loop frequency must be used.

Now if we combine the optimum control gain range given an acceptable error rate (around 4%), movement time, ease of use and fatigue, we have the following optimum ranges:

- **Stiffness 0**: control gains [0.1 – 0.5]
- **Stiffness 30**: control gains [0.3 – 0.7]
- **Stiffness 120**: control gain 0.1
- **Stiffness 800**: control gain 0.1
- **Stiffness 1600**: control gain 0.1

In other words, if the stiffness is greater than 120 N.m\(^{-1}\), a control gain equal to 0.1 m.s\(^{-1}\).N\(^{-1}\) will give the optimal performance for movement time and error rate. For stiffness below 120 N.m\(^{-1}\), this value can be increased up to 0.7 m.s\(^{-1}\).N\(^{-1}\) as the stiffness gets closer to 0. Higher stiffness accommodates a device with a small operating range.

Using Figure 9 or Equation 7, the selected control gain value should be checked for a compatible operating range given the device space constraints, the desired spring stiffness and device display size (with the display diagonal considered as the maximum target distance). If Equation 7
results in an operating range which cannot be accommodated by the device, it suggests that the operating range is too constrained. However, the control gain can be increased to accommodate, but it may degrade performance.

As an example, consider selecting the best control gain for the Nintendo Wii Nunchuck rate control device. The Nunchuck has a 2D elastic joystick with a 20mm operating range and a 60 N.m⁻¹ stiffness. With a maximum target distance of 300 mm, Equation 7 finds an operating range of 30 mm and a control gain of 0.7 m.s⁻¹.N⁻¹. Considering the wide optimum control gain range for this stiffness, we can increase control gain without hurting performance to fall within the 20 mm operating range of the Nunchuck.

CONCLUSIONS AND FUTURE WORK

Thus far, tuning elastic device parameter settings has been guided by inspiration and small pilot experiments. With this study we provided a systematic analysis of not only stiffness, but also its interaction with control gain. We have shown that control gain is a critical factor when tuning rate control devices and demonstrated how the stiffness impacts the optimum range of control gains. With optimum control gain values, we found that elastic and isometric devices can perform equally well. However, operating range and its effect on user fatigue is another important factor to consider. Considering the stiffness, operating range and control gain, we provide the first guidelines for choosing stiffness, and control gain for rate control devices. The results of our experiment give a base line for comparisons involving rate control devices and for experimentation with non-linear functions. As future work, we plan to begin this work by investigating the optimal design of non-linear functions for rate control devices.

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