Hybrid Metaheuristics

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Motivations

- Large scale multi-objective optimization problems in different industrial domains (Telecommunications, Genomics, Transportation and Logistics, Engineering design, ...).

- Problem size more and more important (combinatorial explosion) et/ou Delays more and more reduced.

**Objective**: Efficient Modelling and Solving of Large (Multi-objective) Combinatorial Optimization Problems

\[
\begin{align*}
(POC) & \quad \min f(x) \quad x \in S \\
(POM) & \quad \begin{cases} 
\min f(x) = (f_1(x), f_2(x), \ldots, f_n(x)) & n \geq 2 \\
\text{s.c. } x \in S
\end{cases}
\end{align*}
\]
Optimization Algorithms

- Exact methods are useless for large problems
- Metaheuristics are efficient (lower bound, best known results, …).
- Lack of theoretical results (applicable in a real context).
Solution-based metaheuristics

- “Improvement” of a solution
- Exploitation oriented
  - Based on the descent
  - Exploration of the neighborhood (intensification)

- Local search
- Escape from local optima: Simulated Annealing, Tabu Search, ILS, VNS, GLS, …
Population-based metaheuristics

- “Improvement” of a population
- Exploration oriented
  - Solutions distributed in the search space
  - Recombination is explorative
Direct recombination

Evolutionary Algorithms: GA, GP, ES, EA
Scatter Search
Indirect recombination: Ant colonies

- Artificial ants: Dorigo (1992)
- Imitate the cooperative behavior of ant colonies to solve optimization problems
- Use very simple communication mechanism: pheromone
  - Olfactive and volatile substance
  - Evolution: evaporation, reinforcement

During the trip, a pheromone is left on the ground. The quantity left depends on the amount of food found. The path is chosen accordingly to the quantity of pheromones. The pheromone has a decreasing action over time.

**Ant : Solution construction, Pheromone update**
Particle Swarm

- Population based stochastic metaheuristic
- Dr. Eberhart and Dr. Kennedy (1995)
- Inspired by social behavior of bird flocking or fish schooling

A nature-inspired process

- Particles fly through the problem space
- Flight = add a velocity to the current position
- Social adaptation of knowledge
- Particles follow the current optimum particles («follow the bird which is nearest to the food »)

Particle: Evaluate the velocities, Flight, Update the bests
Estimation of Distribution Algorithm

- Based on the use of (unsupervised) density estimators/generative statistical models
- Idea is to convert the optimization problem into a search over probability distributions
- The probabilistic model is in some sense an explicit model of (currently) promising regions of the search space

- Initialize a probability model Q(x)
- Do
  - Create a population of points by sampling from Q(x)
  - Evaluate the objective function for each point
  - Update Q(x) using selected population and f() values
- While termination criterion not reached
Population-based incremental Learning (PBIL)

- Initial distribution \( D=(0.5, \ldots, 0.5) \)
- Iteration:
  - Generation of the population
  - \( X_i = 1 \) if \( r < D_i \) (\( r \) uniform in \([0,1]\))
  - \( X_i = 0 \) else
  - Evaluate and sort the population
  - Update the distribution

\[
D = (1 - \alpha)D + \alpha \cdot X_{\text{best}}
\]

S. Baluja, R. Caruana. Removing the Genetics from the Standard Genetic Algorithm. ICML’95
Other probability models

- Mutual Information Maximization for Input Clustering (MIMIC) regards pairwise dependances
  

- Bayesian Optimization Algorithm (BOA) for multivariate dependances
  
Main search components

- Representation (Encoding)
- Objective function
- Variation Operators, Neighborhood
- Diversification, Exploration
- Intensification, Exploitation
- Hybridization
- Parallellism and Distribution
Why Hybridation?

- Equilibrate exploration / exploitation: Gain in performance, Robustness
- Taxonomy: Grammar – Large variety of classified methods

Hybrid Metaheuristics

- Low-level
  - Relay
  - TeamWork

- High-level
  - Relay
  - TeamWork

Hierarchical

Exact  Approximate  Global  Partial  General  Specialist

Resolution  Search space  Problems  Application  uniformity

Design issues (Hierarchical)

- **Low-level / High-level**
  - High-level: Different methods are self-contained.

- **Relay / Teamwork**
  - Relay: Pipeline fashion.
  - Teamwork: Parallel cooperating agents.
Low-Level Relay Hybrid

- **LRH**: Hybrids in which a given metaheuristic is embedded into a single-solution metaheuristic.

- **Example**: Local search embedded in Simulated annealing (Markov chain explores only local optima) [Martin et Otto 92]
Low-Level Teamwork Hybrid

- **LTH**: A optimization method is embedded into a population-based method.

- **Examples**:  
  - **Greedy crossover** [Grefenstette 85], **local search for mutation** [Fleurant 94, Chu 97] (lamarckian, baldwin, …) in GAs.  
  - **Local search in Ant colonies** [Taillard 97, Talbi et al. 99], **Scatter search** [Van Dat 97], and **Genetic programming** [O'Reilly 95].

![Diagram](image)
**High-Level Relay Hybrid**

- **HRH**: Self-contained optimization methods are executed in a sequence.

- **Example**:
  - GA + Simulated annealing [Mahfoud 95]. GA + Tabu search [Talbi 94].
  - ES + Local search [Nissen 94]. Simulated annealing + GA [Lin 91]
High-Level Teamwork Hybrid

- **HTH**: Parallel cooperating self-contained algorithms.

- **Example**:
  - Island model for GAs [Tanese 87], Genetic programming [Koza 95], Evolution strategies [Voigt 90].
  - Tabu search [Rego 96], Simulated annealing [De Falco 95], Ant colonies [Mariano 98], Scatter search [Van Dat et al. 99].

Topology, Frequency of migration, which individuals to migrate ?, Which individuals to replace ?, ...
High-Level Teamwork Hybrid

Example (meta exact): Cooperation between multi-objective genetic algorithm and a single objective Branch & Cut algorithm (sub-problem for Vehicle Routing Problem) [Jozefowiez, Talbi 04].
**Global versus Partial**

- **Partial**: Problem is decomposed in sub-problems. Each algorithm is dedicated to solve one sub-problem.

- **Examples**:
  - HTH: Tabu search (Vehicle routing) [Taillard 93], Simulated annealing (placement of macro-cells) [Casoto et al. 86].
  - HTH: GA (Job-shop scheduling) [Husbands et al. 90].

**Problem Specific**

- **Problem**: Build a global viable solution
  - Sub-problem: Genetic algorithm
  - Sub-problem: Genetic algorithm
  - Sub-problem: Genetic algorithm
  - Tabu search
  - Simulated annealing

**Synchronisation**
Specialist versus General

**Specialist**: Algorithms which solve different problems.

**Examples**:

- **HTH**: GA + Tabu (QAP) [Talbi et al. 98]
- **HRH**: GA + (SA | NM | GA | AC) [Krueger 93][Shahookar et al. 90][Abbattista et al. 95]

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**HTH**

Parallel tabu search to solve the QAP

- tabu
- tabu
- tabu

**Frequency memory**

**Communication medium**

**Initial solutions**

**Genetic algorithm to solve a different optimization problem (diversification task)**

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**HRH**

Optimizing the parameters of another heuristic

**Genetic algorithm**

**Simulated annealing**

**Noisy method**

**Genetic algorithm**

**Ant colonies**
Robot path planning

(European project PAPAGENA)

Search Agent SEARCH (local optima)

\[\min_{M \in S_s} F_s(M, \tau) = \begin{cases} 0 & \text{If a direct movement exist from } m_i \text{ to } \tau \ (i < a) \\ \min_{i=a-1} \| \tau - m_i \| & \text{Else} \end{cases}\]
Robot path planning

Diversification agent EXPLORE

\[\min_{M \in S_e} F_e(M) = \max_{M \in S_e} \min_{\lambda_k \in \Lambda} \| m_a - \lambda_k \|\]

\[\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}\]

Set of landmarks

SEARCH (Parallel GA) ↔ EXPLORE (Parallel GA)

Fil d’Ariane (Real robots with Aleph Technologies)
COSEARCH: 3 complementary agents cooperate via an adaptive memory

- **Diversifying Agent**: Solutions in unexplored regions
- **Intensifying Agent**: Promising solutions
- **Search Agent**: Explored space

**Adaptive Memory**
- Explored regions
- Promising regions

COSEARCH for QAP

Diversifying Genetic Algo

Path relinking Intensifier

Adaptive memory

Global Frequencies

Initial Solutions

Elite Solutions

The fitness function refers to the adaptive memory

Solutions being in unexplored regions

Solutions being in promising regions

Pick elite solutions

Local frequency matrix

Initial solution

Best solution found

Multiple Tabu Search

Tabu search

Tabu search

Tabu search

Tabu search
Heuristic approach:
- VLNS: Very Large Neighborhood Search (use of an exact method)
- Generation of different Sub-Problems solved by an exact method

Exact approach:
- Good solutions found by metaheuristics to reduce the visited search space for exact methods

Cooperative Metaheuristic

Upper bound, Partial solutions

Branch and Bound, Cut

Lower bound, Optimal Solution sous-pb, Best neighbor

Constraint programming


Grammar for extended schemes

\[
\begin{align*}
< \text{hybrid method} > & \longrightarrow < \text{design-issues} > < \text{implementation-issue} > \\
< \text{design-issues} > & \longrightarrow < \text{hierarchical} > < \text{flat} > \\
< \text{hierarchical} > & \longrightarrow < \text{LRH} > < \text{LCH} > < \text{HRH} > < \text{HCH} > \\
< \text{LRH} > & \longrightarrow \text{LRH} (< \text{method} > (< \text{method} >)) \\
< \text{LCH} > & \longrightarrow \text{LCH} (< \text{method} > (< \text{method} >)) \\
< \text{HRH} > & \longrightarrow (< \text{method} >+ < \text{method} >) \\
< \text{HCH} > & \longrightarrow \text{HCH} (< \text{method} >) \\
< \text{flat} > & \longrightarrow (< \text{resolution} >, < \text{optimization} >, < \text{function} >) \\
< \text{resolution} > & \longrightarrow \text{exact} | \text{approached} \\
< \text{optimization} > & \longrightarrow \text{global} | \text{partial} \\
< \text{function} > & \longrightarrow \text{general} | \text{specialist} \\
< \text{implementation-issue} > & \longrightarrow \text{sequential} | \text{parallel} < \text{scheduling} > \\
< \text{scheduling} > & \longrightarrow \text{static} | \text{dynamic} | \text{adaptive} \\
< \text{method} > & \longrightarrow < \text{exact} > | < \text{heuristic} > \\
< \text{heuristic} > & \longrightarrow \text{LS} | \text{TS} | \text{SA} | \text{GA} | \text{ES} | \text{GP} | \text{GH} | \text{AC} | \text{SS} | \text{NM} | \ldots < \text{hybrid method} > \\
< \text{exact} > & \longrightarrow \text{B&B} | \text{B&C} | \text{B&P} | \text{PL} | \text{PD} | \text{MS} | \ldots < \text{hybrid method} >
\end{align*}
\]
Example of extended hybrid

HTH (HRH (GH + LTH(GA(LS))))

[Levine 94]
Set Partitioning Problem
Airline crew scheduling
Parallel static implementation

[Braun 90]
Traveling salesman problem
Sequential implementation